Implications of Finite Universe Complexity

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Abstract

We show that if the universe has finite complexity then the laws of physics are time reversible, time is periodic with history eternally repeating, the feasible states of the physical universe are non-arbitrary, and the complexity of the physical universe is constant conditional on knowledge of the laws of physics. Time itself is fundamentally non-unique and defined by our chosen decomposition of the universe into physical states and physics. Furthermore, if the Law of Inertia holds (Newton's 1st Law) then both time and space are periodic. Finite complexity is also a possible explanation for spooky action at a distance.

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1. Main Results

Let us define the universe to be everything that exists. Consistent with empirical evidence to date, we suppose that infinity is a mathematical abstraction that does not exist in nature, and so the universe has finite complexity. By this we mean that a complete description of the universe has finite length. Note that this is a very general restriction and does not constrain space or time to be finite or discrete.

If the universe has finite complexity then it must also be deterministic. If this were not the case then random events would keep injecting random/incompressible information into the universe, causing the complexity of the universe to increase without bound, contrary to our assumption of finite complexity.

Let us discretise time (if it is not already discrete) by considering time steps over an arbitrary fixed interval of time $\epsilon \in \mathbb{R}$ and let the state S_t represent the finite information content of a full description of the physical universe (including any hidden variables) at a single instant in time $t\epsilon$ for $t \in \mathbb{Z}$. Without loss of generality, let us assume $\epsilon = 1$ (which is equivalent to a scaling of our time coordinate by $1/\epsilon$). The passage of time is then the result of a constant physics algorithm P of finite Kolmogorov complexity that is applied to the state S_t to produce the state S_{t+1}.

The state S_t is a binary string encoding a full description of the physical universe at instant $t \in \mathbb{Z}$, and the program P is a binary string representing a computer program in a Turing complete language that when executed on a Turing machine with S_t as an auxiliary input, produces S_{t+1} as output and halts. The passage of time is thus represented by $S_{t+1} = P[S_t]$.

If we denote the Kolmogorov complexity of the physical universe S_t by $K(S_t)$, then finite information content implies that there exists some upper limit $L \in \mathbb{N}$ such that $K(S_t) \leq L$ for all t. The state of the physical universe at any instant can therefore be described in L bits or less. By the pigeonhole principle, there exists a period $T \in \{1, \ldots, 2^{L+1}-1\}$ such that $S_{t+T} = S_t$ because there are only $2^0+2^1+\ldots+2^L = 2^{L+1}-1$

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unique binary strings of maximum length L. By repeatedly applying P to both sides of $S_{t+T} = S_t$ we can see that the period T holds for all $t \in \mathbb{Z}$. Time is therefore periodic with history eternally repeating itself.

A corollary of this result is that $S_t = S_{t+T} = P^T[S_t]$ for all $t \in \mathbb{Z}$ and so P^T is equivalent to the identity algorithm when applied to any feasible state of the physical universe S_t for $t \in \mathbb{Z}$. Thus P is invertible on this domain with $P^{-1} = P^{T-1}$. The laws of physics are therefore time reversible on the domain of feasible physical universe states. This means that the physics operator P is a reversible computation which requires zero energy (Landauer, 1961 [1]).

The physics algorithm P is constant by definition because any time-varying parameters are considered part of the current state of the physical universe S_t. The decomposition of the universe into physical universe states S_t and the constant physics operator P is not unique. So the definition of time itself is not unique because it is defined by how we choose to make this decomposition. The most complex physics algorithm P^+ is one that would work equally well on completely random universe states, namely an algorithm that contains a list of every S_t for $t \in \{0, \ldots, T-1\}$ and simply looks up S_t in the list and returns S_0 if $t = T$ and S_{t+1} otherwise. The least complex physics algorithm P^- is the identity operator I, where we consider the entire history of the universe to be a single physical state S_0 existing at a sole instant $t = 0$ and time ceases to exist. Both P^+ and P^- lack understanding in the sense that they do not compress the information content of the universe. A more natural choice of P is that which maximises our understanding/compression of the entirety of existence under the framework of this paper. This is the P that minimises $\min_t K(P, S_t)$ thus resulting in the least complex description of the entire repeating history of the universe[3].

Given complete knowledge of of physics P the complexity of the physical universe is constant (within additive $O(1)$ terms) due to our ability to apply P to the current state of the universe S_t to obtain S_{t+1}, or apply P^{-1} to S_{t+1} to obtain S_t (S_t is the penultimate state obtained by repeatedly applying P to S_{t+1} until S_{t+1} reoccurs),

$$K(S_t|P) \stackrel{\pm}{=} K(P[S_t]|P) = K(S_{t+1}|P). \tag{1}$$

So our laws of physics, if ever complete, must conserve the conditional Kolmogorov complexity of the physical universe[4].

The Second Law of Thermodynamics states that entropy tends to increase with time, which at first glance would seem to contradict conservation of complexity. However, entropy as measured by this law is a measure of complexity conditional on knowledge of the current state of the physical universe, but ignorant of the laws of physics. With knowledge only of the current state of the physical universe, Kolmogorov complexity increases trivially as follows[5]

$$K(S_{t+1}|S_t) = K(P[S_t]|S_t) \geq O(1) = K(S_t|S_t). \tag{2}$$

Irrespective of whether the laws of physics P are local in nature, the law of conservation of complexity is global. The equilibrium cycle of physical universe states is a result of the laws of physics P as well as the

[3]Our intuitive notion of time is due to the stream of physical information reaching our consciousness. With limited mental bandwidth and processing power this stream of information is compressed in the time dimension by the estimation of a personal physics operator P^* that attempts to minimise $K(S^*_{t^*+1}|P^*S^*_{t^*})$ for the timescales t^* and states of the physical universe $S^*_{t^*}$ of personal concern - thus making classical physics intuitive (as it is similar to P^*). Outside of this natural domain the only intuition that serves us well is our intuition to minimise complexity. It is no surprise that mathematics serves us well here because mathematics is the informationless manipulation of information.

[4]One might speculate that the principle of conservation of energy is a proxy for this condition, given we are only just beginning to unravel the equivalence between information and energy.

[5]The perception of an 'arrow of time' may also be explained by our scale of observation being small relative to the information mixing effects of P, even though complexity is conserved at the scale of the universe. At large scales gravity is likely the direct manifestation of the finite complexity limit on the physical universe that reverses any progress made towards heat death at smaller scales.

finite complexity limit on these states. So the laws of physics only need to conserve complexity when applied to the actual feasible states of the physical universe.

Imagine starting the physical universe with an arbitrary physical state U. As we repeatedly apply the physics operator we shall eventually reach a point where a state repeats with $P^{k+N}[U] = P^k[U]$ and so the equilibrium cycle of physical universe states is $P^{k+i}[U]$ for all $i \in \{0, 1, \ldots, N-1\}$ which may or may not include the arbitrary initial state U. The feasible states of the physical universe are therefore non-arbitrary. They have been shaped globally in time and space by the eternal application of the laws of physics under the constraint of finite complexity to result in a feasible repeating cycle. The current state of the physical universe is pre-destined to conserve complexity globally when acted on by the laws of physics, even if this appears fortuitous or miraculous from our local perspective. This is an explanation for any 'spooky action at a distance' that cannot be explained directly by local application of the laws of physics.

The periodic nature of time requires that after a period of time T we return to the current state S_t. If a random transmission of particles is sent out into space it cannot result in these particles being present anywhere after a period of time T where they were not already present. If they are unaffected by obstacles and do not lose energy (we assume here that Newton's 1st Law is true in our chosen physics P) they must therefore travel to a point where this precise configuration of particles already exists. For a sufficiently complex transmission (given the finite complexity of the physical universe), this point can only be their point of origin. A transmission of particles sent into space will therefore reappear from the opposite direction after a period of time T. This is true irrespective of the direction in which the particles are transmitted and so the shape of space must be symmetrical with respect to the passage of information between any point and itself during an interval of time T. Space is therefore symmetrical and unbounded with uninterrupted travel in any direction ultimately resulting in a return to our starting point in both space and time after a period of time T[6].

References

[1] R. Landauer, Irreversibility and heat generation in the computing process, IBM journal of research and development 5 (3) (1961) 183–191.

[6]We might also pass through our starting point several times prior to time T.